Course Program

✔ 6 lectures on Thursdays
  ❖ First lecture Thursday 17.03 in Room 4I-346
  ❖ 10:15-12:00 (15 minutes break in-between)

✔ 6 exercises sessions on Fridays
  ❖ Exercises must be returned beforehand (will count in final grade)
  ❖ First exercises session 18.03 in Room 4I-346
  ❖ 13:15-15:00

✔ 2 labworks
  ❖ Preliminary exercises (will count in final grade)

✔ Exam
  ❖ 23.05,
  ❖ XX

✔ 5 ov
Course Schedule

1. Introduction and Optical Fibers (17.03)
2. Nonlinear Effects in Optical Fibers (24.03)
3. Fiber-Optic Components I (31.03)
4. Fiber-Optic Components II (07.04)
5. Transmitters and Receivers (05.05)
Lecturers

In case of problems, questions...

✓ Course lecturers
  - G. Genty (5 lectures)  goery.genty@tut.fi
  - F. Manoocheri (1 lectures)  farshid.manoocheri@tkk.fi
Optical Fiber Concept

✔ Optical fibers are light pipes

✔ Communications signals can be transmitted over these hair-thin strands of glass or plastic

✔ Concept is a century old

✔ But only used commercially for the last ~30 years when technology had matured
Why Optical Fiber Systems?

✓ Optical fibers have more capacity than other means (a single fiber can carry more information than a giant copper cable!)
✓ Price
✓ Speed
✓ Distance
✓ Weight/size
✓ Immune from interference
✓ Electrical isolation
✓ Security
Optical Fiber Applications

Optical fibers are used in many areas:

- > 90% of all long distance telephony
- > 50% of all local telephony
- Most CATV (cable television) networks
- Most LAN (local area network) backbones
- Many video surveillance links
- Military
Optical Fiber Technology

An optical fiber consists of two different types of solid glass:

- **Core**
- **Cladding**
- **Mechanical protection layer**

**1970:** first fiber with attenuation (loss) <20 dB/km

**1979:** attenuation reduced to 0.2 dB/km → commercial systems!
Optical Fiber Communication

Optical fiber systems transmit modulated infrared light

Information can be transmitted over very long distances due to the low attenuation of optical fibers
Frequencies in Communications

100 km
10 km
1 km
100 m
10 m
1 m
10 cm
1 cm
1 µm

- wire pairs
- coaxial cable
- waveguide
- optical fiber

- Submarine cable
- Telephone
- Telegraph
- TV
- Radio
- Satellite
- Radar
- Telephone
- Data
- Video

- frequency
- 3 kHz
- 30 kHz
- 300 kHz
- 3 MHz
- 30 Mhz
- 300 MHz
- 3 GHz
- 30 GHz
- 300 THz
Frequencies in Communications

<table>
<thead>
<tr>
<th>Data rate</th>
<th>Optical Fiber: &gt; Gb/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Micro-wave ~10 Mb/s</td>
</tr>
<tr>
<td></td>
<td>Short-wave radio ~100 kb/s</td>
</tr>
<tr>
<td></td>
<td>Long-wave radio ~4 Kb/s</td>
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</table>

Increase of communication capacity and rates requires higher carrier frequencies

Optical Fiber Communications!
Optical Fiber

Optical fibers are cylindrical dielectric waveguides

**Dielectric:** material which does not conduct electricity but can sustain an electric field

![Diagram of optical fiber with core and cladding](image)

Cladding diameter 125 µm
Core diameter from 9 to 62.5 µm

Cladding (pure silica)
Core silica doped with Ge, Al...

Typical values of refractive indices

- **Cladding:** $n_2 = 1.460$ (silica: SiO$_2$)
- **Core:** $n_1 = 1.461$ (dopants increase ref. index compared to cladding)

A useful parameter: fractional refractive index difference $\delta = (n_1 - n_2) / n_1 \ll 1$
Fiber Manufacturing

Optical fiber manufacturing is performed in 3 steps

✓ Preform (soot) fabrication
  ✷ deposition of core and cladding materials onto a rod using vapors of SiCCL₄ and GeCCL₄ mixed in a flame burned

✓ Consolidation of the preform
  ✷ preform is placed in a high temperature furnace to remove the water vapor and obtain a solid and dense rod

✓ Drawing in a tower
  ✷ solid preform is placed in a drawing tower and drawn into a thin continuous strand of glass fiber
Fiber Manufacturing

Step 1

Soot Preform

Burner

Vapor → Fuel

Steps 2&3

Movable Preform Holder

Preform

Furnace

Fiber Drawing

Diameter Monitor

Coating Applicator

Ultraviolet Lamps

Tractor Assembly

Take-Up Spool
Light Propagation in Optical Fibers

- Guiding principle: Total Internal Reflection
  - Critical angle
  - Numerical aperture

- Modes

- Optical Fiber types
  - Multimode fibers
  - Single mode fibers

- Attenuation

- Dispersion
  - Inter-modal
  - Intra-modal
Total Internal Reflection

Light is partially reflected and refracted at the interface of two media with different refractive indices:

✓ Reflected ray with angle identical to angle of incidence
✓ Refracted ray with angle given by Snell’s law

Snell’s law:
\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

Angles \( \theta_1 \) & \( \theta_2 \) defined with respect to normal!

✓ Refracted ray with angle: \( \sin \theta_2 = n_1 / n_2 \sin \theta_1 \)
✓ Solution only if \( n_1 / n_2 \sin \theta_1 \leq 1 \)
Total Internal Reflection

Snell’s law:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

\[ \sin \theta_c = \frac{n_2}{n_1} \]

If \( \theta > \theta_c \) No ray is refracted!

For angle \( \theta \) such that \( \theta > \theta_c \), light is fully reflected at the core-cladding interface: optical fiber principle!
Numerical Aperture

- For angle $\theta$ such that $\theta < \theta_{\text{max}}$, light propagates inside the fiber.
- For angle $\theta$ such that $\theta > \theta_{\text{max}}$, light does not propagate inside the fiber.

**Example:**

$n_1 = 1.47$
$n_2 = 1.46$
$NA = 0.17$

\[ NA = n_1 \sin \theta_{\text{max}} = \sqrt{n_1^2 - n_2^2} \approx n_1 \sqrt{2\delta} \] with $\delta = \frac{n_1 - n_2}{n_1} << 1$

Numerical aperture $NA$ describes the acceptance angle $\theta_{\text{max}}$ for light to be guided.
Theory of Light Propagation in Optical Fiber

✓ Geometrical optics can’t describe rigorously light propagation in fibers

✓ Must be handled by electromagnetic theory (wave propagation)

✓ Starting point: Maxwell’s equations

\[
\nabla \times E = -\frac{\partial B}{\partial T} \quad (1)
\]
\[
\nabla \times H = J + \frac{\partial D}{\partial T} \quad (2)
\]
\[
\nabla \cdot D = \rho_f \quad (3)
\]
\[
\nabla \cdot B = 0 \quad (4)
\]

with

\[
B = \mu_0 H + M \quad : \text{Magnetic flux density}
\]
\[
D = \varepsilon_0 E + P \quad : \text{Electric flux density}
\]
\[
J = 0 \quad : \text{Current density}
\]
\[
\rho_f = 0 \quad : \text{Charge density}
\]
Theory of Light Propagation in Optical Fiber

\[ P(r,t) = P_L(r,t) + P_{NL}(r,t) \]

\[ P_L(r,t) = \varepsilon_0 \int_{-\infty}^{+\infty} \chi^{(1)}(t - t_1) E(r,t_1) dt_1 \]  : Linear Polarization

\[ P_{NL}(r,T) \] : Nonlinear Polarization

\[ \chi^{(1)} \] : linear susceptibility

We consider only linear propagation: \( P_{NL}(r,T) \) negligible
Theory of Light Propagation in Optical Fiber

\[ \nabla \times \nabla \times E(r, t) + \frac{1}{c^2} \frac{\partial^2 E(r, t)}{\partial t^2} = -\mu_0 \frac{\partial^2 P_L(r, t)}{\partial t^2} \]

We now introduce the Fourier transform: \( \tilde{E}(r, \omega) = \int_{-\infty}^{+\infty} E(r, t) e^{i\omega t} dt \)

\[ \frac{\partial^k E(r, t)}{\partial t^k} \Leftrightarrow (i\omega)^k \tilde{E}(r, \omega) \]

And we get: \( \nabla \times \nabla \times \tilde{E}(r, \omega) - \frac{\omega^2}{c^2} \tilde{E}(r, \omega) = +\mu_0 \varepsilon_0 \chi^{(1)}(\omega) \omega^2 \tilde{E}(r, \omega) \)

which can be rewritten as

\[ \nabla \times \nabla \times \tilde{E}(r, \omega) - \frac{\omega^2}{c^2} \left[ 1 + c^2 \mu_0 \varepsilon_0 \chi^{(1)}(\omega) \right] \tilde{E}(r, \omega) = 0 \]

i.e. \( \nabla \times \nabla \times \tilde{E}(r, \omega) - \frac{\omega^2}{c^2} \varepsilon(\omega) \tilde{E}(r, \omega) = 0 \)
Theory of Propagation in Optical Fiber

\[ \varepsilon(\omega) = \left[ n + i \frac{\alpha c}{2\omega} \right]^2 \quad \text{with} \quad n = 1 + \frac{1}{2} \Re \left[ \chi^{(1)}(\omega) \right] \]

\[ \text{and} \quad \alpha = \frac{\omega}{cn(\omega)} \Im \left[ \chi^{(1)}(\omega) \right] \]

\[ \nabla \times \nabla \times \tilde{E}(r, \omega) = \nabla \left( \nabla \cdot \tilde{E}(r, \omega) \right) - \nabla^2 \tilde{E}(r, \omega) = -\nabla^2 \tilde{E}(r, \omega) \]

\[ \nabla \cdot \tilde{E}(r, \omega) \propto \nabla \cdot \tilde{D}(r, \omega) = 0 \]

\[ \nabla^2 \tilde{E}(r, \omega) + n^2 \frac{\omega^2}{c^2} \tilde{E}(r, \omega) = 0 \quad : \text{Helmoltz Equation!} \]
Each component of $E(x,y,z,t) = U(x,y,z)e^{i\omega t}$ must satisfy the Helmholtz equation

$$\nabla^2 U + n^2 k_0^2 U = 0$$

with

\[
\begin{align*}
    n &= n_1 \text{ for } r \leq a \\
    n &= n_2 \text{ for } r > a \\
    k_0 &= \frac{2\pi}{\lambda}
\end{align*}
\]

Note: $\lambda = \omega / c$

Assumption: the cladding radius is infinite

In cylindrical coordinates the Helmholtz equation becomes

\[
\frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} + \frac{1}{r^2} \frac{\partial^2 U}{\partial \phi^2} + \frac{\partial^2 U}{\partial z^2} + n^2 k_0^2 U = 0
\]

with

\[
\begin{align*}
    n &= n_1 \text{ for } r \leq a \\
    n &= n_2 \text{ for } r > a \\
    k_0 &= \frac{2\pi}{\lambda}
\end{align*}
\]
Theory of Light Propagation in Optical Fiber

✓ $U = U(r, \varphi, z) = U(r)U(\varphi)U(z)$
✓ Consider waves travelling in the $z$-direction $U(z) = e^{-j\beta z}$
✓ $U(\varphi)$ must be $2\pi$ periodic $U(\varphi) = e^{-jl\phi}$, $l = 0, \pm 1, \pm 2\ldots$ integer

$U(r, \varphi, z) = F(r)e^{-jl\phi}e^{-j\beta z}$ with $l = 0, \pm 1, \pm 2\ldots$

Plugging into the Helmoltz Eq. one gets:

$$\frac{d^2F}{dr^2} + \frac{1}{r}\frac{dF}{dr} + \left(n^2k_0^2 - \beta^2 - \frac{l^2}{r^2}\right)F = 0$$

with

$$\begin{align*}
n &= n_1 \text{ for } r \leq a \\
n &= n_2 \text{ for } r > a \\
k_0 &= 2\pi / \lambda_0
\end{align*}$$

One can define an effective index of refraction $n_{\text{eff}}$ such that $\beta = \frac{\omega}{c} n_{\text{eff}}$, $n_2 < n_{\text{eff}} < n_1$
A light wave is guided only if \( n_2 k_0 \leq \beta \leq n_1 k_0 \)

We introduce

\[
\kappa = (n_1 k_0)^2 - \beta^2 \\
\gamma^2 = \beta^2 - (n_2 k_0)^2 \\
\kappa^2 + \gamma^2 = k_0^2 \left( n_1^2 - n_2^2 \right) = k_0^2 NA^2 : \text{constant!}
\]

Note: \( \kappa, \gamma \geq 0 \)
\( \kappa, \gamma : \text{real} \)

We then get:

\[
\frac{d^2 F}{dr^2} + \frac{1}{r} \frac{dF}{dr} + \left( \kappa^2 - \frac{l^2}{r^2} \right) F = 0 \text{ for } r \leq a \\
\frac{d^2 F}{dr^2} + \frac{1}{r} \frac{dF}{dr} - \left( \gamma^2 + \frac{l^2}{r^2} \right) F = 0 \text{ for } r > a
\]
The solutions of the equations are of the form:

\[ F_l(r) = J_l(\kappa r) \quad \text{for } \rho \leq a \]

\[ J_l : \text{Bessel function of 1}^{\text{st}} \text{ kind with order } l \]

\[ F_l(r) = K_l(\gamma r) \quad \text{for } \rho > a \]

\[ K_l : \text{Modified Bessel function of 1}^{\text{st}} \text{ kind with order } l \]

with

\[ \kappa^2 = (n_1k_0)^2 - \beta^2 \]

\[ \gamma^2 = \beta^2 - (n_2k_0)^2 \]

\[ \kappa^2 + \gamma^2 = k_0^2\left(n_1^2 - n_2^2\right) = k_0^2NA^2 : \text{constant!} \]
Examples

\[ l=0 \]

\[ F(r) \propto \begin{cases} 
J_0(\kappa r) & \text{for } r \leq a \\
K_0(\gamma r) & \text{for } r < a 
\end{cases} \]

\[ l=3 \]

\[ F(r) \propto \begin{cases} 
J_3(\kappa r) & \text{for } r \leq a \\
K_3(\gamma r) & \text{for } r < a 
\end{cases} \]
Boundary conditions at the core-cladding interface give a condition on the propagation constant $\beta$ (characteristics equation):

The propagation constant $\beta$ can be found by solving the characteristic equation:

$$
\begin{bmatrix}
J'_i(K) + \frac{K'_i(\Gamma)}{\Gamma K_i(\Gamma)}
\end{bmatrix} \times \begin{bmatrix}
\frac{n_1^2}{n_2^2} K J_i(K) + \frac{K'_i(\Gamma)}{\Gamma K_i(\Gamma)}
\end{bmatrix} = \frac{l^2 \beta_{lm}^2}{n_2^2 k_0^2} \left[\frac{1}{K^2} + \frac{1}{\Gamma^2}\right]^2
$$

with $K = a\kappa$ and $\Gamma = a\gamma$

For each $l$ value there are $m$ solutions for $\beta$. Each value $\beta_{lm}$ corresponds to a particular fiber mode.
Number of Modes Supported by an Optical Fiber

- Solution of the characteristics equation $U(r, \phi, z) = F(r)e^{-j\phi}e^{j\beta_{lm}z}$ is called a mode, each mode corresponds to a particular electromagnetic field pattern of radiation.

- The modes are labeled $LP_{lm}$.

- Number of modes $M$ supported by an optical fiber is related to the $V$ parameter defined as

$$V = ak_0 NA = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2}$$

- $M$ is an increasing function of $V$!

- If $V < 2.405$, $M=1$ and only the mode $LP_{01}$ propagates: the fiber is said to be Single-Mode.
Number of Modes Supported by an Optical Fiber

✓ Number of modes well approximated by:

\[ M \approx \frac{V^2}{2}, \text{ where } V^2 \approx \left( \frac{2\pi a}{\lambda} \right)^2 \left( n_1^2 - n_2^2 \right) \]

Example:

- \( 2a = 50 \ \mu m \)
- \( n_1 = 1.46 \)
- \( V = 17.6 \)
- \( \delta = 0.005 \)
- \( M = 155 \)
- \( \lambda = 1.3 \ \mu m \)

✓ If \( V < 2.405 \), \( M = 1 \) and only the mode LP\(_{01}\) propagates: Single-Mode fiber!
Examples of Modes in an Optical Fiber

$\lambda = 0.6328 \, \mu m \quad a = 8.335 \, \mu m \quad n_j = 1.462420 \quad \delta = 0.034$

- $LP_{01}$
- $LP_{11}$
- $LP_{22}$
- $LP_{31}$
Examples of Modes in an Optical Fiber

\[ \lambda = 0.6328 \, \mu m \quad a = 8.335 \, \mu m \quad n_f = 1.462420 \quad \delta = 0.034 \]
The propagation constant of a given mode depends on wavelength \([\beta(\lambda)]\)

The cut-off condition of a mode is defined as \(\beta(\lambda)-k_0^2 n_2^2= \beta(\lambda)-4\pi^2 n_2^2/\lambda^2=0\)

There exists a wavelength \(\lambda_c\) above which only the fundamental mode LP\(_{01}\) can propagate

\[
V < 2.405 \iff \lambda_c = \frac{2\pi}{2.405} n_1 a \sqrt{2\delta} = 1.84 a n_1 \sqrt{\delta}
\]

or equivalently \(a = \frac{2.405}{2\pi} \frac{\lambda_c}{n_1 \sqrt{\delta}} = 0.54 \frac{\lambda_c}{\sqrt{n_1 \delta}}\)

**Example:**
- \(2a = 9.2 \mu m\)
- \(n_1 = 1.4690\)
- \(\delta = 0.0024\)
- \(\lambda_c \approx 1.2 \mu m\)
Single-Mode Guidance

In a single-mode fiber, for wavelengths $\lambda > \lambda_c \sim 1.26 \, \mu m$ only the LP$_{01}$ mode can propagate.
Mode Field Diameter

The fundamental mode of a single-mode fiber is well approximated by a Gaussian function

\[ F(\rho) = Ce^{-\left(\frac{\rho}{w_0}\right)^2} \]

where \( C \) is a constant and \( w_0 \) the mode size

A good approximation for the mode size is obtained from

\[ w_0 = a \left( 0.65 + \frac{1.619}{V^{3/2}} + \frac{2.879}{V^6} \right) \text{ for } 1.2 < V < 2.4 \]

\[ w_0 = \frac{a}{\sqrt{\ln(V)}} \text{ for } V > 2.4 \]
Types of Optical Fibers

Step-index single-mode

- Cladding diameter: 125 µm
- Core diameter: from 8 to 10 µm
- Refractive index profile: $\delta = 0.001$

$\delta = 0.001$
Types of Optical Fibers

Step-index multimode

Cladding diameter from 125 to 400 µm
Core diameter from 50 to 200 µm

Refractive index profile

\[ \delta = 0.01 \]
Types of Optical Fibers

Graded-index multimode

- Core diameter: from 50 to 100 μm
- Cladding diameter: from 125 to 140 μm
- Refractive index profile

n
n

r
Signal attenuation in optical fibers results from 3 phenomena:
- Absorption
- Scattering
- Bending

Loss coefficient: \( \alpha \)

\[
P_{\text{Out}} = P_{\text{In}} e^{-\alpha L}
\]

\[
10 \log_{10} \left( \frac{P_{\text{Out}}}{P_{\text{In}}} \right) = -\alpha L \frac{10}{\ln(10)} = -4.343\alpha L
\]

\( \alpha \) depends on wavelength

For a single-mode fiber, \( \alpha_{\text{dB}} = 0.2 \text{ dB/km} @ 1550 \text{ nm} \)
Scattering and Absorption

- **Short wavelength: Rayleigh scattering**
  - induced by inhomogeneity of the refractive index and proportional to \(1/\lambda^4\)

- **Absorption**
  - Infrared band
  - Ultraviolet band

- **3 Transmission windows**
  - 820 nm
  - 1300 nm
  - 1550 nm
Macrobending Losses

Macrobending losses are caused by the bending of fiber

- Bending of fiber affects the condition $\theta < \theta_c$
- For single-mode fiber, bending losses are important for curvature radii $< 1$ cm
Microbending Losses

Microbending losses are caused by the rugosity of fiber.

Micro-deformation along the fiber axis results in scattering and power loss.
Attenuation: Single-mode vs. Multimode Fiber

Light in higher-order modes travels longer optical paths

Multimode fiber attenuates more than single-mode fiber
Dispersion

✓ What is dispersion?
   - Power of a pulse travelling though a fiber is dispersed in time
   - Different spectral components of signal travel at different speeds
   - Results from different phenomena

✓ Consequences of dispersion: pulses spread in time

✓ 3 Types of dispersion:
   - Inter-modal dispersion (in multimode fibers)
   - Intra-modal dispersion (in multimode and single-mode fibers)
   - Polarization mode dispersion (in single-mode fibers)
In a multimode fiber, different modes travel at different speeds, causing temporal spreading (inter-modal dispersion).

- Inter-modal dispersion limits the transmission capacity.
- The maximum temporal spreading tolerated is 1/2 bit period.
- The limit is usually expressed in terms of bit rate x distance.
Dispersion in Multimode Fibers (Inter-modal)

✓ Fastest ray guided along the core center

✓ Slowest ray is incident at the critical angle

\[ AT = T_{SLOW} - T_{FAST} \]

with \( T_{FAST} = \frac{L_{FAST}}{v_{FAST}} \) and \( T_{SLOW} = \frac{L_{SLOW}}{v_{SLOW}} \)

\[ v_{FAST} = v_{SLOW} = \frac{c}{n_1} \]

\[ L_{FAST} = L \]

\[ L_{SLOW} = \frac{L}{\cos \theta} = \frac{L}{\sin \left( \frac{\pi}{2} - \theta \right)} = \frac{L}{\sin \theta_c} = \frac{n_1}{n_2} L \]

\[ AT = \frac{n_1}{c} L - \frac{n_1^2}{n_2 c} L = \frac{n_1^2}{n_2 c} L \left[ 1 - \frac{n_2}{n_1} \right] = \frac{n_1^2}{n_2} L \delta \]
Dispersion in Multimode Fibers

If bit rate $= B \, b/s^{-1}$

We must have $\Delta T < \frac{1}{2B}$

i.e. $\frac{L n_1^2}{c n_2} \delta < \frac{1}{2B}$

or $L \times B < \frac{cn_2}{2n_1^3}$

Example: $n_1 = 1.5$ and $\delta = 0.01 \rightarrow B \times L < 10 \, \text{Mb/s}^{-1}$

Capacity of multimode-step index fibers $B \times L \approx 20 \, \text{Mb/s} \times \text{km}$
Dispersion in graded-index Multimode Fibers

- Fast mode travels a longer physical path
- Slow mode travels a shorter physical path

Temporal spreading is small

Capacity of multimode-graded index fibers $B \times L \approx 2 \text{ Gb/s} \times \text{km}$
Intra-modal Dispersion

- In a medium of index $n$, a signal pulse travels at the group velocity $v_g$ defined as:

$$v_g = \frac{d\omega}{d\beta} = \left( -\frac{\lambda^2}{2\pi c} \frac{d\beta}{d\lambda} \right).$$

- Intra-modal dispersion results from 2 phenomena:
  - Material dispersion (also called chromatic dispersion)
  - Waveguide dispersion

- Different spectral components of signal travel at different speeds

- The dispersion parameter $D$ characterizes the temporal pulse broadening $\Delta T$ per unit length per unit of spectral bandwidth $\Delta \lambda$: $\Delta T = D \times \Delta \lambda \times L$

$$D_{\text{Intra-modal}} = \frac{d}{d\lambda} \left( \frac{1}{v_g} \right) = -\frac{\lambda^2}{2\pi c} \frac{d^2 \beta}{d\lambda^2} \text{ in units of ps/nm} \times \text{km}$$
Material Dispersion

✓ Refractive index $n$ depends on the frequency/wavelength of light

✓ Speed of light in material is therefore dependent on frequency/wavelength

Input pulse, $\lambda_1$

Input pulse, $\lambda_2$
Material Dispersion

Refractive index of silica as a function of wavelength is given by the **Sellmeier Equation**

\[
n(\lambda) = \sqrt{1 + \frac{A_1\lambda^2}{\lambda_1^2 - \lambda^2} + \frac{A_2\lambda^2}{\lambda_2^2 - \lambda^2} + \frac{A_3\lambda^2}{\lambda_3^2 - \lambda^2}}
\]

with \( A_1 = 0.6961663, \ \lambda_1 = 68.4043 \text{ nm} \)
\( A_2 = 0.4079426, \ \lambda_2 = 116.2414 \text{ nm} \)
\( A_3 = 0.8974794, \ \lambda_3 = 9896.161 \text{ nm} \)
Material Dispersion

\[ v_g = \left( -\frac{\lambda^2}{2\pi c} \frac{d\beta}{d\lambda} \right)^{-1} = \frac{c}{n - \lambda \frac{dn}{d\lambda}} \]

\[ \Delta T = L\Delta\lambda D = L\Delta\lambda \frac{d}{d\lambda} \left( \frac{1}{v_g} \right) = \frac{L}{c} - \lambda\Delta\lambda \frac{d^2n}{d\lambda^2} \]
Material Dispersion

\[ D_{\text{Material}} = -\frac{\lambda}{c} \frac{d^2n}{d\lambda^2} \text{ (units: ps/nm} \times \text{km)} \]

\[ D_{\text{Material}} = 0 \text{ @1.27 } \mu\text{m} \]
Waveguide Dispersion

- The size $w_0$ of a mode depends on the ratio $a/\lambda$: $w_0 = a\left(0.65 + \frac{1.619}{\nu^{3/2}} + \frac{2.879}{\nu^6}\right)$

- Consequence: the relative fraction of power in the core and cladding varies

- This implies that the group-velocity $\nu_g$ also depends on $a/\lambda$

$$D_{\text{Waveguide}} = \frac{d}{d\lambda} \left( \frac{1}{\nu_g} \right) = \frac{\lambda}{2\pi^2 nc} \frac{d}{d\lambda} \left( \frac{\lambda}{w_0^2} \right)$$

where $w_0$ is the mode size
Total Dispersion

\[ D_{\text{Intra-modal}} = D_{\text{Material}} + D_{\text{Waveguide}} \]

Wavelength (nm)

<table>
<thead>
<tr>
<th>Dispersion [ps/(nm km)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>-10</td>
</tr>
</tbody>
</table>

\[ D_{\text{Intra-modal}} < 0: \text{normal dispersion region} \]
\[ D_{\text{Intra-modal}} > 0: \text{anomalous dispersion region} \]

Waveguide dispersion shifts the wavelength of zero-dispersion to 1.32 μm
Tuning Dispersion

- Dispersion can be changed by changing the refractive index
- Change in index profile affects the waveguide dispersion
- Total dispersion is changed

Single-mode Fiber: $D=0 \ @ \ 1310 \text{ nm}$
Dispersion shifted Fiber: $D=0 \ @ \ 1550 \text{ nm}$
Dispersion Related Parameters

\[ \beta = \frac{\omega}{c n_{\text{eff}}} \]

\[ \frac{1}{v_g} = \frac{d\beta}{d\omega} = \beta_1 : \text{group delay in units of s/km} \]

\[ D_{\text{Intra-modal}} = \frac{d}{d\lambda} \left( \frac{1}{v_g} \right) = \frac{d\beta_1}{d\lambda} = \frac{d\beta_1}{d\omega} \frac{d\omega}{d\lambda} = \beta_2 \left( -\frac{2\pi c}{\lambda^2} \right) \]

\[ \beta_2 : \text{group velocity dispersion parameter in units of s}^2/\text{km} \]
Optical fibers are not perfectly circular

In general, a mode has 2 polarizations (degenerescence): $x$ and $y$

Causes broadening of signal pulse

$$\Delta T = L \left( \frac{1}{v_{gx}} - \frac{1}{v_{gy}} \right) \approx D_{\text{Polarization}} \sqrt{L}$$
Effects of Dispersion: Pulse Spreading

Total pulse spreading is determined as the geometric sum of pulse spreading resulting from intra-modal and inter-modal dispersion.

\[ \Delta T = \sqrt{\Delta T_{\text{Intermodal}}^2 + \Delta T_{\text{Intra-modal}}^2 + \Delta T_{\text{Polarization}}^2} \]

Multimode Fiber: \[ \Delta T = \sqrt{(D_{\text{Inter-modal}} \times L)^2 + (D_{\text{Intra-modal}} \times \Delta \lambda \times L)^2} \]

Single - Mode Fiber: \[ \Delta T = \sqrt{(D_{\text{Intra-modal}} \times \Delta \lambda \times L)^2 + (D_{\text{Polarization}} \times \sqrt{L})^2} \]

Examples: Consider a LED operating @ .85 µm \( \Delta \lambda = 50 \) nm after \( L = 1 \) km, \( \Delta T = 5.6 \) ns

- \( D_{\text{Inter-modal}} = 2.5 \) ns/km
- \( D_{\text{Intra-modal}} = 100 \) ps/nm×km

Consider a DFB laser operating @ 1.5 µm \( \Delta \lambda = .2 \) nm after \( L = 100 \) km, \( \Delta T = 0.34 \) ns!

- \( D_{\text{Intra-modal}} = 17 \) ps/nm×km
- \( D_{\text{Polarization}} = 0.5 \) ps/√km
Effects of Dispersion: Capacity Limitation

Capacity limitation: maximum broadening $< 1/2f$ a bit period

$$\Delta T < \frac{1}{2B}$$

For Single-Mode Fiber, $\Delta T \approx LD_{\text{intra-modal}} \Delta \lambda$

(neglecting polarization effects)

$$\Rightarrow LB < \frac{1}{2D_{\text{intra-modal}} \Delta \lambda}$$

Example: Consider a DFB laser operating @ 1.55 µm

$\Delta \lambda = 0.2$ nm

$D = 17$ ps/nm×km

$LB < 150$ Gb/s ×km

If $L=100$ km, $B_{\text{Max}}=1.5$ Gb/s
Advantage of Single-Mode Fibers

✓ No intermodal dispersion
✓ Lower attenuation
✓ No interferences between multiple modes
✓ Easier Input/output coupling

Single-mode fibers are used in long transmission systems
Summary

Attractive characteristics of optical fibers:

- Low transmission loss
- Enormous bandwidth
- Immune to electromagnetic noise
- Low cost
- Light weight and small dimensions
- Strong, flexible material
Summary

- **Important parameters:**
  - $NA$: numerical aperture (angle of acceptance)
  - $V$: normalized frequency parameter (number of modes)
  - $\lambda_c$: cut-off wavelength (single-mode guidance)
  - $D$: dispersion (pulse broadening)

- **Multimode fiber**
  - Used in local area networks (LANs) / metropolitan area networks (MANs)
  - Capacity limited by inter-modal dispersion: typically 20 Mb/s x km for step index and 2 Gb/s x km for graded index

- **Single-mode fiber**
  - Used for short/long distances
  - Capacity limited by dispersion: typically 150 Gb/s x km