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# **Instruction Manual of Modelling of Trap Detectors Using Matlab**

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## 2. Introduction

This instruction manual describes the use of models for reflectance and internal quantum efficiency of trap detectors. The models are valid in the range of spectrum between 260 nm and 950 nm wavelengths. The trap detector in this document is a reflection trap detector containing three silicon photodiodes described more precisely in [1]. Please notice that the main attention in this instruction manual is on the practical use of the models. Instead of describing carefully all theoretical or technical aspects, original sources are referred.

The models added to this manual have been realised by using Matlab 2008B. Please notice that principles described in this manual are valid also for every other practical realisation of the same theoretical model. Specific instructions for using the spreadsheet file trapMeasurements.xls and matlab file trapMain.m are given in Appendix A.

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[1] P. Kärhä, *Trap Detectors and Their Applications in the Realisation of Spectral Responsivity, Luminous Intensity and Spectral Irradiance Scales*, Thesis for the degree of Doctor of Technology, HUT, Espoo, 1997, 92 p.

### 3. Definitions

In this document, the following definitions apply:

UV – wavelength range lower than 400 nm;

VIS – wavelength range higher than 400 nm.

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## 4. Reflectance model

The theoretical background and the mathematical formulation of the reflectance model are described in [1]. Following steps have to be made to model the reflectance of the trap detector:

- measurement of reflectances of the trap detector;
- fitting the curve of the reflectance model through the measured values;
- evaluation of the results and uncertainty.

### 4.1. Measurement of reflectances

The measurement setup is described in Figure 1. The laser power is measured by Trap 1 and the power of the reflected beam by Trap 2. Ratio of these measured powers is equal to the reflectance of Trap 1, assuming that the reflectances are approximately equal.

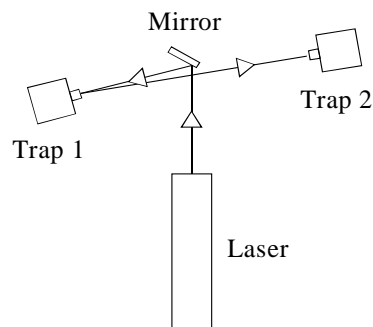


Figure 1. Measurement of the reflectance of trap detector Trap 1.

Distances between traps, mirror and laser should be chosen optimal to minimise the tilt angle of Trap 1 and to avoid divergence of the reflected beam out of the view of Trap 2. Spatial filter can be used to improve the beam quality, if needed.

The reflectances of trap detectors are measured at least at three laser wavelengths. If the laser light is linearly polarised, measurements with both s- and p-polarisation should be conducted. The average of these measurements will be used for reflectance modelling. Measurements at lower wavelengths are preferred because of higher reflectance in UV. Wavelengths that are used for the trap responsivity measurements in the cryogenic radiometer should be included. Suggested wavelengths are given in Table 1.

**Table 1.** Suggested wavelengths for trap detector reflectance measurements.

	Type of laser	Nominal wavelength, nm	Order of magnitude of trap detector reflectance, %
1.	Argon-Ion	458	0.50
2.		476	0.45
3.		488	0.40
4.		514	0.35
5.	HeNe	633	0.25

#### 4.2. Fitting the reflectance model

The measured reflectance values are used as input data for the reflectance model calculation software. The absolute difference between the measured and modelled reflectance values is calculated for each measured wavelength. The reflectance model has one degree of freedom - the average thickness  $t$  of the silicon dioxide layer on the photodiodes. The fitting of the model curve is realised by changing the parameter  $t$  to minimise the sum of squares of calculated differences. For the Hamamatsu type photodiodes considered here the published values of thickness typically vary between 24 and 30 nm. The refractive indices of silicon and silicon dioxide, needed for the model, are taken from [2], [3] and [4], and also added to this manual in TrapModel.xls file.

#### 4.3. Evaluation of the results and uncertainty

The uncertainty of the model has been analysed in [5]. Components that should be included in the uncertainty budget of the measurements are listed in Table 2. The agreement between measured and modelled reflectances has to be within expanded uncertainty limits of the reflectance measurements. The uncertainties of the modelled reflectance values  $u_{mod}(\lambda)$  should be estimated each time by trial of maximising the model curve fluctuations within fixed uncertainty limits at measured wavelengths.

[2] G. E. Jellison Jr., "Optical functions of silicon determined by two-channel polarization modulation ellipsometry," *Opt. Mater.* 1, 41-47 (1992).

[3] I. H. Malitson, "Interspecimen comparison of refractive index of fused silica," *J. Opt. Soc. Am.* 55, 1205-1209 (1965).

[4] E.D. Palik, *Handbook of optical constants of solids*, Academic Press, New York, pp. 547-569 1985.

[5] A. Haapalinna, P. Kärhä, E. Ikonen, "Spectral reflectance of silicon photodiodes," *Appl. Opt.* 37. 729-732 (1998).

**Table 2.** Uncertainty components of the reflectance measurements.

Uncertainty component	Comment	Example, %
1. Repeatability of measurements	Type A, standard deviation of measurements	0.20
2. Calibration of measured trap detector	Type B, from calibration certificate	0.04
3. Calibration of reference trap detector	Type B, from calibration certificate	0.04
4. Spatial nonuniformity of traps	Type B	0.01
5. Linearity of the trap detectors	Type B	0.01
6. Current measurement	Type B	0.01
7. Tilt of the measured trap detector	Type B, depends on the setup	0.05

Uncertainty of the reflectance modelling will be included in the uncertainty budget of modelled responsivities of trap detector. The relative standard uncertainty of the responsivity due to the reflectance measurements and modelling  $u_r$  is calculated as

$$u_r(l) = r(l) u_{\text{mod}}(l)$$

where  $r(l)$  is the modelled reflectance of the trap detector at the wavelength  $l$ .

If the trap detector is used at the same laser wavelengths where reflectances are measured, the modelled values should be used in the responsivity calculations instead of measured values.

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## 5. Internal quantum efficiency model

Theoretical background and mathematical formulation of the internal quantum efficiency (IQE) model is described in [6]. Modelling in visible region is based on work reported by L. Werner [7]. A good reference for UV modelling is [8]. In order to combine the two models, the wavelength around 385 nm at which the models intersect is located. For wavelengths shorter than this wavelength, the UV model is used; and for longer wavelengths longer the visible model is used. The program gives an error message if it the intersection point is absent and then uses 385 nm as a default.

Following steps have to be made to model the IQE of the trap detector:

- reflectance of trap detector has to be measured and modelled;
- responsivity of the trap has to be measured against cryogenic radiometer;
- responsivity of the trap has to be measured against pyroelectric radiometer in UV, if the responsivity will be modelled also in UV;
- fitting the model curve through the measured values;
- evaluation of the results and uncertainty.

### 5.1. Input data

Measurement and modelling of the reflectance are described in Chapter 4 of this document. Measurement of the trap detector responsivity against cryogenic radiometer is described in Instruction Manual of Cryogenic Radiometer Measurements. For modelling, responsivity values from the calibration certificates of trap detectors are used. Relative responsivity values in UV can be accepted if absolute measurement results are not available.

### 5.2. Fitting the IQE model

Because of practical reasons, actual fitting of the IQE model is done by fitting the calculated responsivity curve through the measured responsivity values. Responsivity curve is calculated from IQE and reflectance models. There are seven parameters that can be changed in the used IQE model. Parameters are listed in Table 3. Two of them,  $B$  and

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[6] T. Kùbarsepp *Optical Radiometry Using Silicon Photodetectors*, Thesis for the degree of Doctor of Technology, HUT, Espoo, 1999.

[7] L. Werner, J. Fischer, U. Johannsen, and J. Hartmann, "Accurate determination of the spectral responsivity of silicon trap detectors between 238 nm and 1015 nm using a laser-based cryogenic radiometer," *Metrologia* 37, 279-284 (2000).

[8] T. Kùbarsepp, P. Kärhà, and E. Ikonen, "Interpolation of the spectral responsivity of silicon photodetectors in the near ultraviolet," *Appl. Opt.* 39, 9-15 (2000).



$DE$ , describe increase of quantum yield of silicon in UV and have influence only at wavelengths lower than 375 nm.

The fitting of the responsivity curve through the measured responsivity values is done by changing IQE model parameters to minimise the sum of squares of absolute differences between measured and modelled responsivities.

**Table 3.** Description of IQE model parameters.

	Symbol	Description	Example value	Unit
1.	$P_f$	Collection efficiency at SiO <sub>2</sub> /Si interface	0.962	-
2.	$T$	Depth in the photodiode at which $P_f=1$	$0.26 \cdot 10^{-6}$	m
3.	$P_r$	Collection efficiency in the rear region of photodiode	0.997	-
4.	$D$	Depth at which $P_f$ has linearly decreased to the value of $P_r$	$13 \cdot 10^{-6}$	m
5.	$d$	Thickness of the photodiode	$320 \cdot 10^{-6}$	m
6.	$B$	Scaling factor, includes direct transition-matrix element and doping-condition variations	$1.75 \cdot 10^6$	m <sup>-1</sup>
7.	$DE$	Correction for the absorbed photon energy	0.11	eV

### 5.3. Evaluation of the results and uncertainty

The uncertainty of the IQE model is estimated in [6]. The uncertainty components that should be included in the uncertainty budget are given in Table 4. The agreement between measured and modelled responsivities has to be within expanded uncertainty limits. Results of the modelling should be compared with earlier results. If big differences are observed, the reason for the difference has to be explained.

**Table 4.** Uncertainty components of modelled responsivities.

	Uncertainty component	Comment	Example, %
1.	Calibration of trap detector against cryogenic radiometer	Type B, from calibration certificate	0.04
2.	Reflectance modelling	Type B, from reflectance measurements	0.04
3.	Calibration of trap detector against pyroelectric radiometer	Type B, included only for UV wavelengths	0.35
4.	Uncertainty of the IQE model	Type B, depends on wavelength	0.01 - 0.1

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## 6. Reporting Results

Report of the modelling results has to be issued at least in computer file format as internal calibration certificate. Following data has to be included in the report:

- input data
  - measured reflectances;
  - reflectance setup parameters (tilt angle, distances, beam size);
  - responsivities from calibration certificates;
- modelled reflectance values;
- modelled internal quantum efficiency and responsivity values;
- optimised parameters for model functions;
- uncertainty budgets for
  - reflectance measurement;
  - modelled responsivity;
- comparison with previously modelled values, if available.

Modelled reflectance and responsivity values should be preferably reported with 1 nm wavelength intervals.

## 7. Records

The model files for traps are stored in the following folder:

`\\metrology\webpages\quality\radiom\Trap Responsivities\`

Naming convention for subfolders is initials of the modeller followed by year, e.g. "TD2013". The subfolder will contain the result sheets for all traps modelled, and a folder containing latest versions of files trapMeasurements.xls and trapMain.m. The results are further reported in an internal calibration certificate.

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## 8. Intercomparisons and model verifications

The mathematics used for extrapolating reflectances have been compared with calculations done by previous Excel model. All measurements agree well within uncertainties.

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## Appendix A: User Guide for TrapModel.xls

Modelling is based on two files, *trapMeasurements.xls*, which is used as input for measurements and output for results and *trapMain.m* which is used for calculations.

### *trapMeasurements.xls*

First sheet (Measurements) has three parts. First part is used for reflectance measurements, second for responsivity measurements in visible region (370 nm – 950 nm) and last for responsivity measurements in ultraviolet region (235 nm – 400 nm).

Responsivity measurements in VIS and UV regions should overlap so that correction coefficient between these two can be calculated. Measurements under 235 nm can't be used and there can't be empty lines between measurements!

Second sheet [*n*(Si)] includes refractive index *n*(Si) and extinction coefficient *k*(Si) of silicon, which are needed in modelling.

Running *trapMain.m* adds another sheet to this file (Model), where results are written.

### *trapMain.m*

This Matlab file is used for actual modelling. User must update lines 10-24 to match corresponding measurement file. In addition modelled trap detector must be given name (line 7).

### Running model

#### *trapMeasurements.xls*

- add measured wavelengths, reflectances and uncertainties to three adjacent columns
- add measured wavelengths and responsivities in visible region to next two adjacent columns
- add measured wavelengths and responsivities in uv region to last two adjacent columns
- save the file with a new name (Matlab will overwrite old data)
- close the file (Matlab can't write to open file)

#### *trapMain.m*

- open Matlab
- find *trapMain.m* by changing Current Directory
- open file by writing "edit *trapMain.m*" to command window or by double clicking it
- write detector name to line 7 (ex. `name = 'UVFR-8';`)
- write measurement file path and file name to line 10 (ex. `file = 'C:\Documents and Settings\mttsaari\My Documents\MATLAB\uvfr8_2009.xls';`)

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- write measurement cells to lines 13, 17, 21 and 24. Cells should be written in form A2:C4. (ex. `Lnm = xlsread(file, 'n(Si)', 'A2:C722');`).
  - save file and press F5, select Run trapMain.m from debug menu or write trapMain in command window.
  - program takes ~30 seconds to run. There should be three windows open after program is complete. There is also confirmation in command window.
  - check that responsivity models are overlapping near 380 nm from figure 2. If not, change line 32 to include more or less pyro measurements. This value must be smaller than last pyro measurement.

#### Detailed description of model

First user must update lines 10-24 to match corresponding measurements file. In addition modelled trap detector must be given name (line 7). Line 27 is used to change distance between results (usually 1 nm).

Measured values are split into different variables for each wavelength and measurement except pyro measurements, which are kept as a matrix, where first column contains wavelengths and second measurements.

Line 32 determines how far into visible region uv measurements are used for uv responsivity modelling. Lines 34 and 35 defines uv and visible regions. Lines 38-41 are used to interpolate  $n(\text{Si})$  and  $k(\text{Si})$  over region from beginning of uv to the end of visible.

Lines 60-68 are used to calculate reflectance and it's limits and standard deviation. `reflectanceModel` function is based on [1].

Lines 70-78 are used to fit uv responsivity measurements to corresponding visible region measurement by changing correction coefficient `pyroCorrect`.

Lines 87-119 are used to determine six variables needed to model responsivity by minimizing the sum of squares of differences between modelled and measured values.